The values that will satisfy (1) and (3) are easily found to be $x=0,\ \pm\frac{1}{2}$ and $y=0,\ \pm\frac{1}{2}\sqrt{(-1)},\ \pm\frac{1}{2}$; (1) and (4), $x=0$, and $y=0$; (2) and (3), $x=0, -\frac{1}{8}$, and $y=0, -\frac{1}{16}$; (2) and (4), $x=0$, 8, and $y=0$, 4.

$\therefore\ x=0,\ 8,-\frac{1}{8},\pm\frac{1}{2}$, and $y=0,\ 4,\ -\frac{1}{16},\pm\frac{1}{2},\pm\frac{1}{2}\sqrt{(-1)}$.

Solved with like results by *J. F. W. SCHEFFER*. Also solved by *J. A. CALDERHEAD, H. W. DRAUGHON, A. L. FOOTE, J H. GROVES, and H. C. WHITAKER.*

28. Proposed by P. C. CULLEN, Meade, Nebraska.

A man and a boy get n dollars for digging potatoes, the man can dig them as fast as the boy can pull the vines, but the man can pull vines m times as fast as the boy can dig them. Divide the money.

I. Solution by A. L. FOOTE. C. E., Merrick, New York; P. H. PHILBRICK, C. E., Lake Charles, Louisiana; and J. A. CALDERHEAD, B. Sc., Limaville, Ohio.

Let x be the cost of all the digging, and y the cost of pulling the vines; then $x+y=n \ldots (1)$. Now by the conditions the man can perform the effect x, while the boy performs the effect y. Also the man can do my, while the boy does x. Therefore we have $x : my :: y : x$, or $x^2=my^2 \ \ldots (2)$.

$\therefore\ x=y\sqrt{(m)} \ldots (3)$. Substituting in (1) we have

$$y=\frac{n}{1+\sqrt{(m)}},\ \text{and}\ x=\frac{n\sqrt{(m)}}{1+\sqrt{(m)}}.$$

II. Solution by J. H. DRUMMOND, LL. D, Portland, Maine.

The problem is indeterminate unless we assume the man's rate of digging and of pulling vines bears the same ratio to each other as the boys. Let $x=$the man's time for pulling vines and px, his time of digging; and $y=$ boy's time for pulling vines and py his time of digging. Then $\frac{1}{x}$ and $\frac{1}{px}=$ man's rate of digging and pulling vines, respectively, and $\frac{1}{y}$ and $\frac{1}{py}$, boy's rate.

Then $\frac{1}{px}=\frac{1}{y}$, and $\frac{1}{x}=\frac{m}{py}$. Hence $y=px$ and $y=\frac{mx}{p}=px$. Hence $p^2=m$, and $p=\sqrt{m}$

$\therefore\ y=x\sqrt{m}$. $\frac{1}{x}+\frac{1}{px}=$man's work in same time boy works, $\frac{1}{y}+\frac{1}{py}$, or $\frac{1}{x\sqrt{m}}+\frac{1}{mx}$. Then $\frac{1}{x}+\frac{1}{x\sqrt{m}}+\frac{1}{x\sqrt{m}}+\frac{1}{mx}=$what both would do in same time, or $\frac{m+2\sqrt{m}+1}{mx}$.

Then $n\left(\frac{m+\sqrt{m}}{mx}\right)\div\frac{m+2\sqrt{m}+1}{mx}=$man's part$=\frac{n\sqrt{m}}{1+\sqrt{m}}$, and

$$n\left(\frac{\sqrt{m}+1}{mx}\right)\div\frac{(1+\sqrt{m})^2}{mx}=\text{boy's part}=\frac{n}{1+\sqrt{m}}.$$

III. Solution by J. H. GROVE, Howard College, Brownwood, Texas.

Let $d=$the amount of work digging a certain distance, and $p=$the

amount of work pulling vines the same distance. Also let x=the amount of money the man received and, y=the amount of money the boy received. Since the amounts of money are to each other as the amounts of work done.

$\therefore x:y :: d:p$; but $x:y :: mp:d$.

$\therefore mp:d :: d:p :: x:y. \quad \therefore m=\frac{d^2}{p^2}=\frac{x^2}{y^2}$.

$\therefore x=y\sqrt{m}$. But since $x+y=n$. $\therefore x=n-y$.

$\therefore y\sqrt{m}=n-y. \quad \therefore y=\frac{n}{1+\sqrt{m}}$, and $x=\frac{n\sqrt{m}}{1+\sqrt{m}}$.

Also solved by *P. S. BERG, C. W. M. BLACK, J. K. ELLWOOD, M. A GRUBER, F. P. MATZ, C. E. WHITE, J. F. W. SCHEFFER, H. C. WHITAKER*, and *G. B. M. ZERR.*

26. Proposed by ALVIN E. SCHMIDT, Winesberg, Ohio.

Show that $abc>(a+b-c)(a+c-b)(b+c-a)$ unless $a=b=c$.

I Solution by P. S. BERG, Apple Creek, Ohio.

$$a^2>a^2-(b-c)^2$$
$$b^2>b^2-(a-c)^2$$
$$c^2>c^2-(a-b)^2$$

Multiplying together the corresponding members of these inequalities, $a^2b^2c^2>(a+b-c)^2(a+c-b)^2(b+c-a)^2$.

$\therefore abc>(a+b-c)(a+b-c)(b+c-a$.

II. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Such examples can be proved either by beginning with known principles and ending with the example, or beginning with the example and reducing it to known truths. I will use the latter method.

If $abc>(b+c-a)(a+b-c)(c+a-b)$, then $abc>ab(a+b)+bc(a+c)+ac(a+c)-2abc-a^3-b^3-c^3$ by multiplication, or $a^3+b^3+c^3+3abc>ab(a+b)+bc(b+c)+ac(a+c)$, but $a^3+b^3+c^3>3abc$. Hall and Knight's Algebra, or easily proved. Hence, *a fortiori*, $2(a^3+b^3c+{}^3)>ab(a+b)+bc(b+c)+ac(a+c)$, but this is true, Hall and Knight's Algebra, page 210.

Hence, the original proposition is true.

Also elegantly solved by *B. F. BURLESON, F. P. MATZ*, and *G. B. M. ZERR.*

PROBLEMS.

36. Proposed by J. A. CALDERHEAD, B. Sc., Superintendent of Schools, Limaville, Ohio.

Resolve $(x^2+y^2)(x^2+z^2)(y^2+z)$ into the sum of two squares.

37. Proposed by H. M. CASH, Gibson, Ohio.

The area of the segment of a circle$=c$, and radius$=r$. Find height of segment.

38 Proposed by F. M. SHIELDS, Coopwood, Mississippi.

A man sold 2 horses and a mule for \$286.90. On the first horse he gained as much per cent. as the horse cost dollars, and gained $\frac{5}{8}$ as much per cent. on the second horse as the first, and he loses \$9.10 on the mule. His net gain was \$86.90. What was the cost and selling price of each?